1 Overview and numerical issues

1.1 Three summation algorithms

\[\text{sum} \leftarrow 0\]
\[\text{for } i \leftarrow 1 \text{ to } n \text{ do}\]
\[\text{sum} \leftarrow \text{sum} + x_i\]
\[\text{mean} \leftarrow \text{sum}/n\]

Algorithm 1.1: Obvious algorithm for the mean

\[\text{sum} \leftarrow 0\]
\[\text{for } i \leftarrow 1 \text{ to } n \text{ do}\]
\[\text{sum} \leftarrow \text{sum} + x_i\]
\[t \leftarrow \text{sum}/n\]
\[\text{sum} \leftarrow 0\]
\[\text{for } i \leftarrow 1 \text{ to } n \text{ do}\]
\[\text{sum} \leftarrow \text{sum} + (x_i - t)\]
\[\text{mean} \leftarrow t + \text{sum}/n\]

Algorithm 1.2: Less obvious algorithm for the mean

\[\text{sum} \leftarrow 0\]
\[c \leftarrow 0\]
\[\text{for } i \leftarrow 1 \text{ to } n \text{ do}\]
\[y \leftarrow x_i - c\]
\[t \leftarrow \text{sum} + y\]
\[c \leftarrow (t - \text{sum}) - y\]
\[\text{sum} \leftarrow t\]
\[\text{mean} \leftarrow \text{sum}/n\]

Algorithm 1.3: Nonobvious algorithm for the mean (Kahan)

The Kahan summation algorithm, together with analysis and extensions, is described in Klein (2006). An excellent survey of floating-point systems, focusing on elements that matter in the design of computer systems, can be found in Goldberg (1991). Goldberg discusses floating-point formats, notions of relative and absolute error, and the key concept of cancellation. He also gives a useful overview of the IEEE floating-point standards. Overton (2001) examines the basics of floating-point numerical analysis from the perspective of the IEEE standard. This is a delightful (and genuinely useful) book of roughly 100 pages, with the subtitle “Including One Theorem, One Rule of Thumb, and One Hundred and One Exercises.” Of particular note is the discussion of floating point operations on Intel processors, and illustrations of the central ideas of cancellation, conditioning, and stability.
References

